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Approximations of nonlinear transfer characteristic of analog-to-digital converters suitable for correcting their nonlinearity

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Supervisor:	Vladimír Haasz
Supervisor-specialist:	David Slepička
Author:	Petr Suchánek

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Dissertation Thesis was elaborated in presence form of doctoral study at Department of Measurement, Faculty of Electrical Engineering, CTU in Prague.

Aspirant: Ing. Petr Suchánek
Department of Measurement,
Faculty of Electrical Engineering, CTU in Prague
Technická 2, 166 27 Prague 6

Supervisor: Prof. Ing. Vladimír Haasz, CSc.
Department of Measurement,
Faculty of Electrical Engineering, CTU in Prague
Technická 2, 166 27 Prague 6

Supervisor-specialist: Ing. Bc. David Slepička, Ph.D.
Department of Measurement,
Faculty of Electrical Engineering, CTU in Prague
Technická 2, 166 27 Prague 6

Opponents: ....
....
....

The Syllabus was sent out on ....

The defence of the Dissertation Thesis will take place on ....in ....before the commission for dissertation thesis of the field of study Measurement and Instrumentation in the conference room no. ...., Faculty of Electrical Engineering, CTU in Prague.

The Dissertation Thesis is at disposal in the dean's office, Department of Science, Research and Foreign Affairs, Faculty of Electrical Engineering, CTU in Prague, Technická 2, Prague 6.

chairman of the commission for dissertation thesis
of the field of study Measurement and Instrumentation
Faculty of Electrical Engineering, CTU in Prague, Technická 2, Prague 6

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1 State of the art

1.1 Introduction

Current trends in many technical areas indicate that actual and future implementations will be based on digital signal processing techniques. The performance of such techniques is limited by the analogue front-end. One of the limits is specifically non-idealities of analogue-to-digital converters (ADC). It is therefore an essential task to correct ADC's non-idealities.

A real ADC behaves nonlinearly and it has memory effects, or hysteresis, too. This nonlinearity behaviour can arise anywhere in the ADC's circuitry. The description of nonlinearities can provide an insight into the components operation and its influence to the input signal. Some ADC's parameters can be found in datasheets and can be measured by technique called ADC testing. Those ADC's parameters are INL, DNL, ENOB, THD, SNHR, SFDR, SINAD. The most important parameters are INL and THD, which are describing nonlinearity.

The nonlinear behaviour of an ADC is undesired as it brings new artefacts, namely higher harmonic components, to the recorded data, which become therefore deteriorated. Their impact can be described by some parameters of an ADC, the most related are the THD and INL.

There are methods that could remove some of the effects brought in by nonlinearities. Very often used are methods based on look-up tables (LUTs), where the output code from the ADC is a table index. If a multidimensional table is used, also multiple frequency ranges can be captured and then the correction can be done not only for the frequency or narrow frequency range for which the LUT was prepared. In such a case the index in the table is not only the output code but also the difference between the current and previous output code, which gives the signal slope and therefore the frequency of the signal. On contrary to the LUT based methods, there also exist analytical methods. They reconstruct the ADC output signal based on the analytical expression of the nonlinearity. From such expression its inverse is calculated and applied to the distorted output signal in order to obtain the corrected ADC output. For these methods it is essential to know the model and its coefficients. These are obtained during testing or characterisation of the ADC.

There exist two standardised methods for testing and characterisation; the first one is the IEEE Standard for Digitizing Waveform Recorders, IEEE Std 1057 [1] and second one is the IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters, IEEE Std 1241 [2]. Both standards propose sine wave signals, as a natural waveform, for ADC testing. However, signal generators available at the market do not provide sufficiently pure harmonic signal to suit the test requirements. This is especially true when higher frequencies, more than hundreds of kilohertz, are considered. The solution can be a filter based test measurement setup where the test signal is band-pass filtered prior to input the tested device (device under test – DUT). This technique shall ensure, that the test signal exhibits better characteristics than the tested device. If this is not ensured, the insufficient characteristics of the test generator impact of the characterisation of an ADC. Also, some digital techniques like the digital pre-distortion or digital post-correction can be employed. Digital predistortion is used when analog filter would not be feasible, for example due to the complexity of the test signal. Digital post-correction is the possibility, when analog filtering or digital predistortion cannot be used to provide a spectrally pure test signal.

This thesis presents another analytical method to correct for the nonlinearities. It is based on polynomial representation model of the Integral Nonlinearity. From this model an inverse model, also based on a polynomial, is derived. The inverse is then used for correction.

1.2 State of the art

A model of ADC nonlinearity can help in predicting the behaviour of the ADC. There exist many different modelling approaches, a very good and extensive overview is given in paper [3].

1.2.1 ADC non-linearity modelling

Quantization is a process that transforms a range of analog input values into one discrete digital output code. It is inherently non-linear and introduces irreversible error. An ADC can be considered linear if the range (width) of input values does not depend on the input values. When a higher resolution ADC is considered, the quantisation step becomes smaller and the transfer function can be considered as a relation between the input and output in the sense that for a ideal linear device, the transfer function would be a straight line, while for real devices the transfer function deviates from the straight line by non-linearities.

An ideal device would be linear in the meaning that its output y could be mapped to the input x in the sense of

$$y = ax + b \tag{1}$$

meaning that there are neither higher order terms nor delays.

There exist several approaches in describing the non-linearity, where describing actually means a type of a model used in modelling. One modelling method assigns the error to physical imperfections in a particular ADC component [4], for example the quantizier or the sample-hold unit. Another way of describing the ADC is as a black box model and analyzing the general input–output relationship. This is the conceptional approach in this work. Apart from non-linearity, the behaviour of an ADC shows memory effects and is well worked out in [18], [5], [6]. A model with memory effect is, however, not subject of this work.

Further on, the nonlinearity depends also on the input test signal frequency because of frequency dependence of input circuitry, and amplitude because of possible dificulties in measuring a signal very close to the reference voltage.

The reason, for which modelling of ADC's non-linearities is important can be more, for example: a) the end users can estimate digital output behaviour if they know the error function (model) of the ADC; b) providing the error function with all its dependencies, the end users can correct for the non-linearity by using the inverse function.

1.2.2 ADC non-linearity measurement

There exist many ways to accomplish measurement of the ADC nonlinearity. To list them, a harmonic signal can be used, as well as exponential stimulus [7], [8], DC offset with small variations [9] or noise [10], [11]. The first one listed (harmonic signal stimulus) is most frequently used one.

The methods for ADC or digitizers testing based on the harmonic signal [1], [2] require spectrally-pure testing signal on the ADC input. However, it is necessary to ensure the spectral purity (THD) and SINAD of the testing signal to be at least 10 dB greater than the THD and SINAD caused by the imperfections of a tested digitizer [1].

Unfortunately, signal generators which are available at the market – for example HP VXI E1445, Agilent 33120A, Agilent 33220 or Agilent 33250 – do not provide sufficiently pure harmonic signal to suit the testing of 12-bit to 16-bit ADCs in the frequency range from hundreds of kilohertz to tens of megahertz [12], [13]. The THD of commercially produced generators achieves only about −50 dB in this frequency range. Such a signal is sufficient only for testing of digitizers with effective number of bits ENOB < 8. Therefore, filtering of the test signal is proposed to obtain the desired spectrum components. However, the filtering method can eliminate neither phase noise nor the spurious frequencies; it only attenuates the spectrum components that are out of the specified band-pass area. On one hand, the IEEE Std 1241 [2] also recommends usage of filters, but does not specify requirements to their parameters. On the other hand, the IEEE Std 1057 [1] proposes to use data fitting (to eliminate the input signal impurities) for parameters evaluation.

Correct attenuation, impedance matching and ground loops are also common issues that can degrade signal quality as it was shown and modelled in [27]. They manifest by increased noise in a frequency limited range or induce spurious components. In such cases reflections occur and disturbance is brought by the ground loops. A useful recommendation is to use isolation transformers if such situations occur, or even preventively.

1.2.3 ADC non-linearity correction

Characterisation and testing of ADCs is important in many aspects, for example ADC post-correction, during which the parameters of an ADC are improved based on digital signal processing methods. In general, the correction is aimed to restore the transfer function back into the ideal straight line. In a situation when the non-linearity is analytically expressed, corection methods means searching for an inverse on the non-linear function.

The correction is a means how to compensate for the errors brought by the ADC into the recorded signal. Correction can be conducted in either a corrective way, when there is a term added to the output value, or by a model inversion. The correction is a typical model for a look-up tables, where the output code from the ADC is a table index, and the content of the table are corrective terms. Often, for preserving the memory utilisation, there are the correction terms saved in lower data width than is the nominal output codewidth of the ADC. Such a table would be a static model. A dynamic model with a LUT is done by adding another dimension to the table, where the index is the difference between the current and previous output code, which gives the signal slope and therefore the frequency of the signal. The correction methods based on LUT are suitable for real-time corrections [14], [15] and many others.

Dynamic nonlinear models, which involve dynamic behaviour and not only nonlinearity, can be also described by the Hammerstein Wiener, or a joint Hammerstein-Wiener models. They combine a model of dynamic in the form of a linear transfer function and the nonlinearity by a nonlinear function [15].

2 Aim of the thesis

The aim of this work is to derive a method for correcting the ADC nonlinearity. It requires to find an inverse function of ADC nonlinearity. This work should focus on approximations that use output ADC data (recorded data) in the amplitude spectrum representation as approximation's input. The approximation will be expressed by an analytical function with the aim that its inverse function can be used to correct ADC non-linearity.

To fulfil the aims the work the following tasks have to be carried out:

- Get used with and adopt methods for dynamic parameters measurement of ADC with higher resolution (from 12-bits to 16-bits) on higher frequencies (the range from several megahertz to tens of megahertz) to obtain input data for ADC non-linearity modelling.
- Analyse approximation functions to describe the nonlinearity behaviour in the ADC input–output relationship.
- Implement the methods that approximate the non-linearity behaviour of the transfer function of an ADC. An analysis of common polynomials, Chebyschev polynomials and Fourier series will precede the implementation. The input data for all approximation methods will be the spectrum of ADC output data generated by a sinewave signal.
- Evaluate the performance of the approximations from the perspective of accuracy, noise sensitivity and the complexity of calculations. The evaluation will be carried out both in simulations and experimental measurement.
- Derive inverse functions from the approximation functions describing the input–output relationship of an ADC.
- Use simulated ADC output signal to verify the performance of the nonlinearity correction.

- Experimental validate the ability of the approximations for correction of ADC non-linearity.

3 Proposed methods

3.1 The novel method – Direct polynomial invertion

The calculation of inverse common polynomial will always yield another polynomial as well. There can be shown that the inverted polynomial coefficients b_k are related to a_k coefficients of the original polynomial.

The next section presentes an analytical method to obtain the inverse transfer function. The inverse function allows to calculate the input before it was distorted by the ADS's nonlinearity. Important to notice is that the proposed inversion provides a range limited solutions, meaning that the inversion error is within a reasonable small values only on a restricted domain [16].

The model of ADC INL(n) nonlinearity can described by means of the common polynomials

$$\mathrm{INL}(n) \ = \sum_{h=0}^{H_{\max}} a_k x^k(n) \tag{2}$$

where a_k are the coefficients of the nonlinearity up to the maximum order, $H_{\max}$ is the highest harmonic component considered. Index h in summation starts from zero because all even order nonlinearity coefficients induce a DC level as well as all odd nonlinearity coefficients induce a contribution to the first order component.

Because the function exposed to invertion has to be monotonic the transfer function TF(n) of an ADC is constructed based on the INL(n) in (2) as

$$\mathrm{TF}(n) = n + \mathrm{INL}(n) \tag{3}$$

where n is the ADC code.

Let's in all text assume a general order $H_{\max}$, which will be later on showed only up to the third order of the nonlinearity. Then the transfer function from (3) can be expressed as

$$\mathrm{TF}(n) = y = \sum_{k=1}^{H_{\max}} a_k x^k(n) \tag{4}$$

For this case, let's propose that the inverted transfer function TF^{-1} will also be of the same order and defined as

$$\mathrm{TF}^{-1}(y) = \sum_{k=1}^{H_{\max}} a_k x^k(n) \tag{5}$$

Substituting (4) into (5) yields out a composition $\mathrm{TF}^{-1}(\mathrm{TF}(n)) = n$

$$\mathrm{TF}^{-1}(\mathrm{TF}(n)) = n = \sum_{h=1}^{H_{\max}} b_h \left[\mathrm{TF}(n)\right]^h = \sum_{h=1}^{H_{\max}} b_h \left[\sum_{h=1}^{H_{\max}} a_h n^h\right]^h \tag{6}$$

The general expression (6) changes, when specified $H_{\max} = 3$ leads to

$$\begin{aligned}\mathrm{TF}^{-1}(\mathrm{TF}(n)) &= \sum_{h=1}^{3} b_h \left[\sum_{h=1}^{3} a_h n^h\right]^h = \\ &= b_1(a_1 n + a_2 n^2 + a_3 n^3) \\ &\quad + b_2(a_1 n + a_2 n^2 + a_3 n^3)^2 \\ &\quad + b_3(a_1 n + a_2 n^2 + a_3 n^3)^3 = n\end{aligned}$$

Two methods are derived in the thesis to find the inverted polynomial coefficients b_k follow.

3.2 Post-correction by Direct polynomial inverse – Method 1

Direct inversion stands for a method where coefficients of inverted polynomial $\mathrm{TF}^{-1}(y)$ are calculated directly from (7) in an easy approximate way from the coefficients of the original polynomial. This solution does not provide a least square error inversion but is very easy to deduce.

From (7) it can be seen, that the maximum term involved is of the $(H_{\max})^2$ equations of $H_{\max}$ unknowns, thus over determined system. From this set of equations, only $H_{\max}$ starting from the lowest order, are selected and evaluated to provide unknown b_i coefficients.

Equating the coefficients in (7) of the same power of n on the right and left side shows that the terms (9) to (16) should equal to zero.

$$\begin{aligned}
\mathrm{TF}^{-1}(\mathrm{TF}(n)) = {} & (b_1 a_1)n + \\
& +(a_2 b_1 + a_1^2 b_2)n^2 + \\
& +(a_3 b_1 + 2a_1 a_2 b_2 + a_1^3 b_3)n_3 + \\
& +(a_2^2 b_2 + 2a_1 a_3 b_2 + 3a_1^2 a_2 b_3)n^4 + \\
& +(2a_2 a_3 b_2 + 3a_1 a_2^2 b_3 + 3a_1^2 a_3 b_3)n^5 + \\
& +(a_3^2 b_2 + 6a_1 a_2 a_3 b_3 + a_2^3 b_3)n^6 + \\
& +(3a_2^2 a_3 b_3 + 3a_1 a_3^2 b_3)n^7 + \\
& +(3a_2 a_3^2 b_3)n^8 + \\
& +(a_3^3 b_3)n^9 \qquad (7)
\end{aligned}$$

This gives the following $(H_{\max})^2 = 9$ equations for $H_{\max} = 3$ variables b_1, b_2 and b_3.

$$b_1 a_1 = 1 \qquad (8)$$

$$a_2 b_1 + a_1^2 b_2 = 0 \qquad (9)$$

$$a_3 b_1 + 2a_1 a_2 b_2 + a_1^3 b_3 = 0 \qquad (10)$$

$$a_2^2 b_2 + 2a_1 a_3 b_2 + 3a_1^2 a_2 b_3 = 0 \qquad (11)$$

$$2a_2 a_3 b_2 + 3a_1 a_2^2 b_3 + 3a_1^2 a_3 b_3 = 0 \qquad (12)$$

$$a_3^2 b_2 + 6a_1 a_2 a_3 b_3 + a_2^3 b_3 = 0 \qquad (13)$$

$$3a_2^2 a_3 b_3 + 3a_1 a_3^2 b_3 = 0 \qquad (14)$$

$$3a_2 a_3^2 b_3 = 0 \qquad (15)$$

$$a_3^3 b_3 = 0 \qquad (16)$$

Using the three equations in (8), (9) and (10) a system with unique solution is formed. If the other equations were used the system would be over determined and would require more difficult solution. The solution of these 3 equations is

$$b_1 = \frac{1}{a_1} \qquad (17)$$

$$b_2 = -\frac{a_2}{a_1^3} \qquad (18)$$

$$b_3 = \frac{2a_2^2 - a_3 a_1}{a_1^5} \tag{19}$$

It is important to note once again, that solution (17) to (19) is only an approximation, as some terms were neglected. If the original polynomial does not have the linear term a_1, the inversion would not exist.

In case of a_1, a_2 and a_3 normalization the coeficient a_1 is set to $a_1^{\mathrm{norm}} = 1$, and $a_2^{\mathrm{norm}} = a_2/a_1$ and $a_3^{\mathrm{norm}} = a_3/a_1$.

4 Results

The results of the proposed algorithm were verified both in simulations and also on real data.

4.1 Verification using modelled data

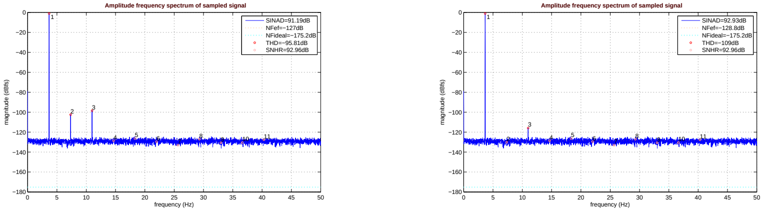
The simulated ADC contained hysteresis that was modelled as

$$y^{\mathrm{hyst}}(x) = \alpha \left(\left(\frac{x}{X_1} \right)^2 - 1 \right) \operatorname{sign}(x) \tag{20}$$

where $y^{\mathrm{hyst}}(x)$ is the contribution of the ADC hysteresis to its output, α is a proportion factor, X_1 is the maximum amplitude of the input signal, and the $\operatorname{sign}(n)$ is a binary function with $+1$ and -1 output values depending on the polarity of the input signal.

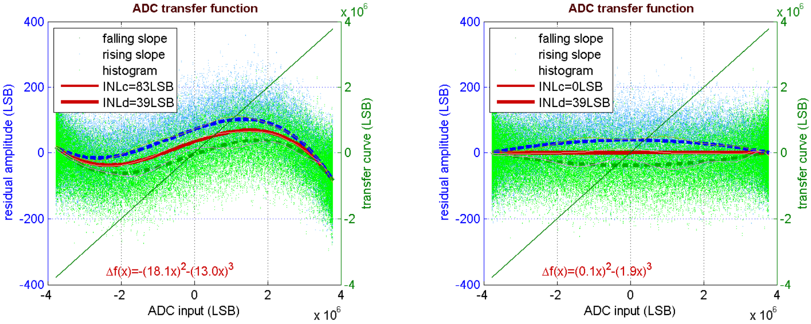
A simulated white noise was also added to the inout signal with $\sigma^2 = 60\,\mathrm{LSB}^2$. This combination resembles the real-situation case.

The Fig. 1c shows a "typical" diagram of the nonlineairity part of the transfer function. The blue and green line is reconstructed from falling and rising slope, respectively, INLd so called "differential-mode" component. The red line is the "common-mode" nonlinearity component INLc [17]. The INLd is caused by hysteresis. The INL nonlineaity is estimated correctly as $\Delta f(x) = -(18.0x)^2 - (13.0x)^3$ LSB. After the correction in Fig. 1d it is seen that the nonlinearity is removed and the only left is the hysteresis. It is correct as the proposed method shall not handle hysteresis, only the nonlinearity.



a) Simulated signal with hysteresis $\alpha = 40\,\mathrm{LSB}$, $\sigma^2 = 60\,\mathrm{LSB}^2$ white noise and input signal phase shift, spectrum before correction

b) Simulated signal with hysteresis $\alpha = 40\,\mathrm{LSB}$, $\sigma^2 = 60\,\mathrm{LSB}^2$ white noise and input signal phase shift, spectrum after correction



c) Simulated signal with hysteresis $\alpha = 40\,\mathrm{LSB}$, $\sigma^2 = 60\,\mathrm{LSB}^2$ white noise and input signal phase shift, transfer function before correction

d) Simulated signal with hysteresis $\alpha = 40\,\mathrm{LSB}$, $\sigma^2 = 60\,\mathrm{LSB}^2$ white noise and input signal phase shift, transfer function after correction

Figure 1 Nonlinearity correction with a complex ADC simulation (hysteresis, noise, phase shift)

4.2 Practical verification on real data

The practical verification is conducted by a Standford Research SRDS360 generator with the output frequency 20.19 kHz and 5.0 Vpp amplitude. Then the signal is routed via a band-pass filter tuned to 20.19 kHz, a 3dB attenuator. The data are recorded by a VXI HP E1430A, which is a 10 MSa/s 23-bit Digitizer. The input range is set to 2 Vpp, the antialiasing filter was switched off and 2 097 152 samples of data was recorded.

The data segments were processed in Matlab, the options for **linsolve()** were set true for upper-triangular and square matrix to internally select the proper solver.

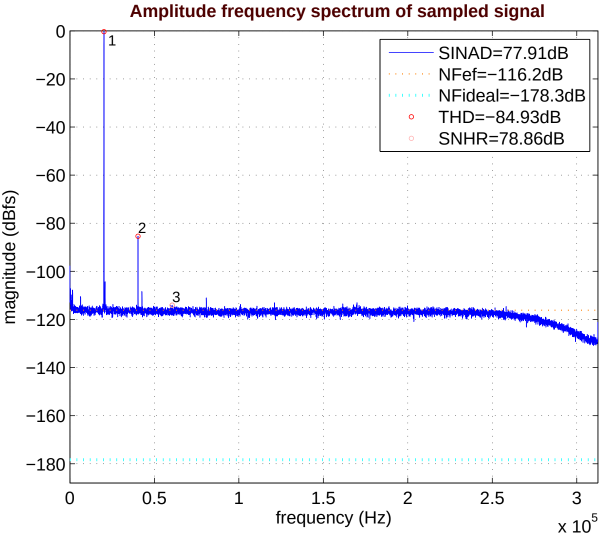
The Fig. 4.2 shows the frequency spectrum of the recorded signal and also the estimation of the transfer function and its nonlinearity based upon a sinewave fitting algorithm.

It is worth mentioning that the noise floor in Fig.2a is 6 dB higher that in Fig. 4.2, although both of them show the spectrum of the recorded signal before correction. The reason is that the latter is one of the segments and is only 1/4 of lenght of the original one. It is also seen from Fig.2b that the attenuation of the second harmonic component is around 15 dB. The third harmonic component is attenuated only minimally, in some of the segments even little bit amplified, by 1–2 dB.

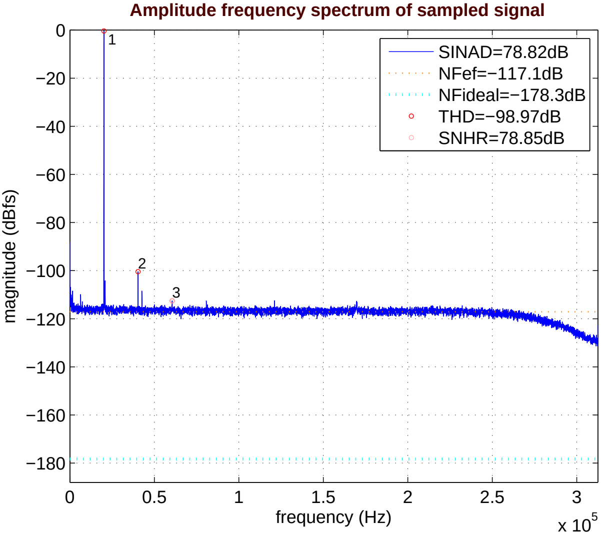
The figures were obtained by applying the Welch's method (also called the periodogram method) for estimating power spectra for each segments before and after correction. The method divides the time signal into successive blocks, forming the periodogram for each block, and averaging (in time). The number of blocks was 30 for the record of $1/4 \cdot 2\ 097\ 152 = 524\ 288$ samples, the factor of overlapping 0.5 and Blackman-Harris window of length 7 was applied to each periodogram. The larger number of periodograms the more averaging and hence the greater spectral stability.

The amplitudes Y_i in the THD evaluation shown in Table 1 were measured from the frequency spectrum and are shown in Table 2.

From the amplitudes measured from the amplitude spectrum and shown in Table 2 the following a_i coefficients in Table 3 are calculated.



a) Real signal, spectrum before correction



b) Real signal, spectrum after correction

Figure 2 A real signal segment (Nr. 2)

Table 1 Improvements in the corrected signal – THD

index	Nr. of set of corr. b_i coefficients	Nr. of corrected segment	THD (dB)	THD improvement (dB)
1	–	–	−85.08	–
2	–	–	−84.97	–
3	–	–	−85.06	–
4	–	–	−85.08	–
5	–	–	−85.01	–
6	1	1	−100.33	15.26
7	1	2	−100.37	15.39
8	1	3	−100.34	15.28
9	1	4	−100.28	15.20
10	2	1	−99.70	14.62
11	2	2	−99.68	14.71
12	2	3	−99.70	14.64
13	2	4	−99.66	14.58
14	3	1	−99.74	14.66
15	3	2	−99.66	14.69
16	3	3	−99.68	14.62
17	3	4	−99.76	14.68
18	4	1	−100.23	15.16
19	4	2	−100.23	15.26
20	4	3	−100.23	15.17
21	4	4	−100.20	15.12

Table 2 Improvements in the corrected signal – Y_i amplitudes

index	Nr. of set of corr. b_i coefficients	Nr. of corrected segment	Y_1 (LSB)	Y_2 (LSB)	Y_3 (LSB)
1	–	–	4 101 835	−224.93	−4.27
2	–	–	4 101 840	−227.87	−6.73
3	–	–	4 101 839	−225.14	−6.95
4	–	–	4 101 834	−224.57	−1.55
5	–	–	4 101 835	−226.71	−5.27
6	1	1	4 101 853	−2.74	−2.48
7	1	2	4 101 861	0.18	0.82
8	1	3	4 101 862	−2.52	1.13
9	1	4	4 101 845	−3.12	−5.30
10	2	1	4 101 853	−0.32	−2.31
11	2	2	4 101 860	2.67	0.93
12	2	3	4 101 861	−0.10	1.23
13	2	4	4 101 845	−0.70	−5.40
14	3	1	4 101 847	0.31	3.18
15	3	2	4 101 855	3.28	5.72
16	3	3	4 101 856	0.53	5.94
17	3	4	4 101 839	−0.07	0.15
18	4	1	4 101 848	−1.67	−0.78
19	4	2	4 101 856	1.27	1.92
20	4	3	4 101 856	−1.46	2.15
21	4	4	4 101 840	−2.05	−3.71

Table 3 Improvements in the corrected signal – a_i coefficients

index	Nr. of set of corr. b_i coefficients	Nr. of corrected segment	a_1 (LSB)	a_2 (LSB)	a_3 (LSB)
1	–	–	4 101 847	−449.86	−17.07
2	–	–	4 101 860	−455.77	−26.91
3	–	–	4 101 860	−450.29	−27.78
4	–	–	4 101 839	−449.14	−6.21
5	–	–	4 101 850	−453.42	−21.10
6	1	1	4 101 861	−5.48	−9.94
7	1	2	4 101 858	0.36	3.28
8	1	3	4 101 858	−5.05	4.51
9	1	4	4 101 861	−6.23	−21.19
10	2	1	4 101 860	−0.64	−9.22
11	2	2	4 101 858	5.34	3.73
12	2	3	4 101 857	−0.19	4.91
13	2	4	4 101 861	−1.40	−21.61
14	3	1	4 101 838	0.62	12.73
15	3	2	4 101 838	6.56	22.86
16	3	3	4 101 838	1.06	23.75
17	3	4	4 101 839	−0.14	0.61
18	4	1	4 101 851	−3.35	−3.12
19	4	2	4 101 850	2.54	7.67
20	4	3	4 101 850	−2.91	8.59
21	4	4	4 101 851	−4.10	−14.85

5 Conclusion

The presented work aimed to derive a new method for correcting the ADC nonlinearity. The most effort was put to polynomials, in the text described as “common polynomials”. A polynomial expression of the inverse function of ADC nonlinearity was found and described and is based on using the ADC output signal spectrum representation. The inversion is expressed in an analytical form.

The correction of the nonlinearities is well described by THD, a measure that describes the values of the higher harmonic components, which are created by the nonlinearity. It is essential for the input test signal to have sufficiently low harmonic distortion, otherwise it influences the results. This topic was partially covered by this thesis.

The results, in simulations, achieved an improvement by the correction as much as 70 dB. In the experimental validation with a real signal the correction achieved approximatelly 15 dB in the THD parameter. It means that the higher harmonic components were suppressed by at least 15 dB.

The following list recapitulates the aims of this thesis stated with references to the relevant chapters in this thesis and author’s publications.

- **Get used with and adopt methods for dynamic parameters measurement for ADCs with higher resolution (from 12-bits to 16-bits) and for higher frequencies (consider the range from several megahertz to tens of megahertz) to obtain input data for modelling.** The work in this filed confirmed the need for a carefull handling with the signal generation and the processing of parameters of the measured ADC. Good experience has been gained in usage of a filtered system.

- **Analyse approximation functions to describe the nonlinearity behaviour in the ADC input–output relationship.** Some figures of merit, as accuracy, sensitivity to noise and performance, of selected approximations were described and presented in papers [30] and [21]. An extensive mathematical description of all three types of INL approximation is provided in the text.

- **Implement the methods that approximate the non-linearity behaviour of the transfer function of an ADC. An analysis of common polynomials, Chebyschev polynomials and Fourier series will precede the implementation. The input data for all**

approximation methods will be the spectrum of ADC output data generated by sine-wave signal. All simulation scripts were implementated in Matlab software.

- **Evaluate the performance of the approximations from the perspective of accuracy, noise sensitivity and the complexity of calculations. The evaluation will be carried out both in simulations and experimental measurement.** The analysys showed that accuracy and noise sensitivity of the three analysed types of approximations (common polynomials, Chebyshev polynomials and Fourier series) of the INL(n) all types of approximations perform comparably for small orders (number of coeficients) of the approximation. For higher number of estimated coefficients (roughly few hundreds) the approximation by common polynomials fails. For large number of coefficients Chebyshev polynomials and Fourier series are further improving and perform similarly. In the case of Fourier series and when the INL(n) from that the Fourier coefficients are calculated is not perfectly repeatable (periodicity is important), the Fourier series approximation features some oscillations at the beginning and end of the INL(n) curve.
- **Derive inverse functions from the approximation functions describing the input–output relationship of an ADC.** The inverse function is derived and the error of neglection of higher terms is conducted. The situation of a real signal is considered and all derivation is carried out for a signal with an offset and general amplitude. For the real case later in the text some simplifications are done. The correction is derived up to the third order nonlinearity, however, the mathematical aparatus is deeply explained so the methodology can easily be extended to higher orders.
- **Use simulated ADC output signal to verify the performance of the nonlinearity correction.** The verification was conducted on simulated data in Matlab. The results are shown and discussed in Chapters **??**, **??** and 4.1. Several different scenarios were simulated for the verification of the proposed algorithm, namely presence of noise in the input signal, hysteresis in the ADC and jitter in sampling. The modelled nonlinearity was assumed approximately THD $= -80$ dB.
- **Experimentaly validate the ability of the approximations for correction of ADC non-linearity.** Practical verification was carried

out on data recorded by a VXI HP E1430A, which is a 10 MSa/s 23-bit Digitizer. The test signal was generated by a SRDS360 generator at the output frequency 20.19 kHz and amplitude 5.0 Vpp, followed by a band-pass filter tuned to 20.19 kHz to filter out higher harmonic components, and then followed by a 3dB attenuator to match the impedance. The input range of the digitizer was set to 2 Vpp, anti-aliasing filter was switched off and 2 097 152 samples was recorded. The results are shown in Chapter 4.2.

5.1 Further work

This chapter contains ideas that came into authors considerations, however, they were not further derived, implemented or validateed, due to their extend and time limitation.

Direct polynomial inverse based on Chebyschev polynomials

The correction by so called direct polynomials has been presented in this thesis by means of the common polynomials. If similar derivation could be done for Chebyshev polynomials, a great deal of work would be done, as the Chebyshev polynomials are othogonal, therefore their coefficients, and they do not suffer by influencing each other (as it happends at the common polynomials).

An approximation based on Taylor series expansion

The same is applicable as in the previous item, as Fourier series are also orthogonal. To fulfill the requirements for the function periodicity, a special care must be taken at both begining and ending part of INL approximation, as it was shown in this thesis.

Time dependency of the coefficients

It would be of interest to run a long term measurement with a digitizer, for instance every one second for tens of minutes. From the recorded data sets a time dependency of the a_i, and consequently b_i coefficients would show the progress in settling down digitizers parameters. If it is possible, a model of such a behaviour could be a subject of a diploma or thesis due to its complexity.

Implement and approve the proposed Method 2

The thesis proposes also an alternative second algorithm to derive the coefficients a_i. This method, however, was not tested, most importantly not on real data. Such an approval shall be conducted and results compared to the Method 1.

Extend the Direct polynomial correction for higher nonlinearity order

Presuming that the nonlinearity of the third order is sufficient enough to carry out basic modelling of correcting methods is straightforward for its simplicity, however, might be misleading for real situations, as a real ADC often exhibit nonlinearities of much higher orders. It would be therefore desirable to extend the correction algorithm to be able to accept more higher harmonic components. Odd harmonic components are very important for their significancy. A reasonable number would be up to 7, while 5 higher components would be the natural first step of extension.

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None.

Patents

None.

Excerpted from WoS

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Summary

In the field of modelling nonlinearities of the transfer functions of the analog-to-digital converters (ADC), a polynomial model is often used. The research conducted in this thesis is focused on so called "common polynomials". These polynomials are also compared with Chebyshev polynomials and Fouries series for the accuracy and noise sensitivity. The novel method for post-correction is proposed in the thesis. It is based on the analytical inversion of the polynomial which, in turn, is used to correct the ADC output signal. This thesis has shown how to derive the correction algorithm up to the third order nonlinearity.

It was sufficiently tested on simulated data and also on a real signal recorded by a 23-bit VXI digitizer. In the theoretical verification the correction achieved around 70 dB on the signal that contained nonlinear distortion THD $= -80\ dB$, while with practical data the correction only reaches approximately 15 dB for a signal with comparable harmonic distortion.

Details of the method are provided and it shall be extended for higher nonlinearity orders also to achieve better performance for real signals, which usually contain distortion of higher orders, too. Some ideas for future work are laid out in the thesis.

Resume

V oblasti modelování nelinearit převodních funkcí analgově číslicových převodníků (AČP) jsou polynomiální modely často používány. Výzkum shrnutý v této práci se zaměřuje na běžné polynomy. Předkládá porovnání přesnosti a šumové odolnosti těchto polynomů s polynomy Čebyshevovými a Fourierovými řadami. Nově představuje metodu korekce výstupu AČP, který je ovlivněn jeho nelinearitou a nabízí postup, jak tuto nelinearitu ve výstupních datech odhalit a do určité míry potlačit.

Metoda je založena na analyticky vyjádřené inverzi převodní charakteristiky, která slouží ke korekci výstupních dat. Metoda byla teoreticky ověřena simulacemi pro zkreslení do třetího řádu a prakticky reálným signálem na 23bitovém VXI digitizéru. V simulacích potlačení nelinearit dosahuje až 70 dB pro simulovane zkreslení na rovni THD $= -80\ dB$, zatímco na reálném signálu převodníku, který má srovnatelné zkreslení, je potlačení pouze cca 15 dB.

Metoda je v práci podrobně popsána a v dalších pracech by měla být rozšířena tak, aby korigovala i složky vyšších harmonických. Tím by i na reálném signálu dokázala dosáhnout lepších parametrů.